

Time-correlation of the solar p-mode velocity signal from GOLF

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Abstract. Since the launch of SOHO, the Solar Heliospheric Orbital Observatory, the helioseismic observations are nearly uninterrupted. The GOLF instrument (A. Gabriel *et al.*, 1997) measures the mean velocity integrated over the disk. The autocorrelation function of this velocity shows two main features: Firstly, the initial decrease of the peak amplitudes is much faster than expected from the width of the most powerful lines and secondly it does not decrease to zero for large times. These two features have been studied using the model of stochastically excited oscillators. The second one can also be understood on the basis of a completely general discussion. We show that the fast initial decrease of the peak amplitudes results from the departure of the mode frequencies from the values predicted by the first order asymptotic theory (the modes are not equidistant) and that the damping time of the modes has only a small influence. The non vanishing amplitudes at large times result either from the presence of a periodic non-stochastic component in the signal or from the stochastic nature of the excitation. Further tests have shown that the second possibility is the right one. This result gives a new argument in favor of the stochastic excitation of solar p-modes. The use of the ACF also suggests a new method to study line profiles which has been tested for radial modes and Lorentz profiles.

Key words: Sun: oscillations

1. Introduction

Interruptions in the data generally do not allow to get the full time autocorrelation function (ACF) of the solar signal. Also because its computation is very time consuming it has never been obtained so far. Today, GOLF has been measuring the solar integrated velocity field without major interruption for more than 2 years. The full ACF can be computed for the first time. It shows features with quasi-periods close to 2 and 4 hours. These values can be readily understood from the first order asymptotic

theory. The large frequency separation $\nu_{n+1} - \nu_n$ which is of the order of 135 μHz leads to the 2 hour period. The 4 hour one appears because (0-2) and (1-3) pairs are separated by about 135/2 μHz . The amplitude variation of these peaks is more surprising and differs for the odd and the even ones. It might be tempting to relate the initial amplitude decrease of the even peaks to the damping time of the most powerful modes. It will be shown in Sects. 5 and 6 that this interpretation is wrong and that the amplitude variation for time lags shorter than about 50 hours is mainly produced by the non-equispacing of the modes (i.e. to the departures from the first order asymptotic approximation). For times larger than 150 hours the amplitude of the envelope stops decreasing, the amplitude of this steady state being related to the length of the data sample used. If just one ACF, corresponding to one data length, is considered, this behavior can possibly be interpreted as the signature of the presence of a non-stochastic component in the signal (see Sect. 4). This is ruled out by a study of the variation of the amplitude at large time lags in term of the length of the data set and therefore that behavior supports the stochastic excitation of the p-modes.

When lines profiles are studied from the power spectrum, the main difficulty arises because, at each Fourier point, the dispersion of power around the mean value remains constant when the observation time increases. On the contrary, if one line is isolated in the power spectrum and the ACF of that portion of the spectrum is then computed, for time lags shorter than a few damping time, the dispersion around the mean value decreases when the observation time increases. This suggests a new independent method to study line profiles which is briefly discussed in Sects. 3 and 7.

The observations are presented in Sects. 2 and 3. In Sect. 4 the properties of the correlation function are analyzed. The behavior of the peak amplitude for short times is studied in Sects. 5 and 6 using the model of stochastically excited oscillators which indeed shows the same behavior at large times. A simple numerical simulation also allows to reproduce the main properties of the observed ACF for short times (Sect. 6).

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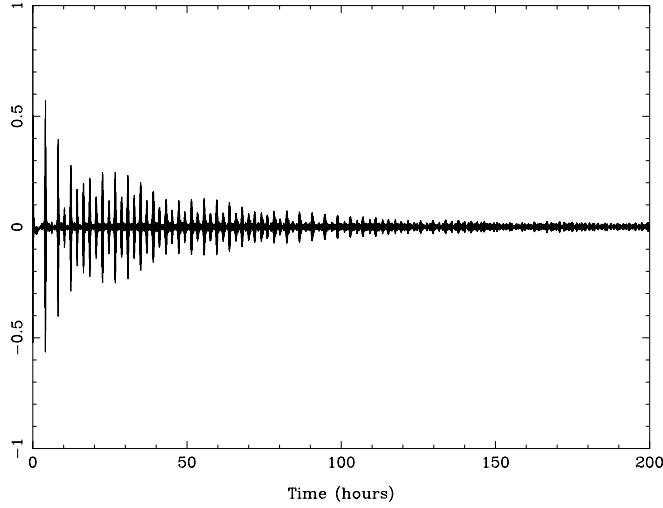


Fig. 1. The normalized ACF of the solar velocity averaged on the solar disk for 550 days of observation, showing the initial fringes.

2. The observations

The observations started April 11, 1996 and are still running. The data are converted in velocity and filtered for the low frequency trends. This final data set is sampled every 40 s and we compute the ACF as

$$R_N(\tau) = \frac{1}{N} \sum_{n=1}^N V(t_n)V(t_n + \tau)$$

where N is the number of samples in the data set of duration T_o and $V(t_n)$ is the observed velocity. The ACF for $T_o = 550$ days is displayed on Figs. 1 and 2 for time lags shorter than 200 and 1500 hours respectively. For short delays, the amplitude variation shows a quasi-periodic pattern, with 2 characteristic periods, close to 2 and 4 h respectively. Those two periods are obviously related to the periodicities found in the power spectrum. The 2 hour period comes in because modes of a given degree are nearly equidistant and separated by about $135 \mu\text{Hz}$. The 4 hour period shows up because the odd degree mode frequencies are about halfway between two successive modes of even degree. The time variation of the amplitude of these peaks is more surprising. The amplitude of the even peaks (those which appear for time delays which are multiple of 4 hours) first decreases quickly with time, much faster than the damping time of the most powerful lines. Later on they show beatings. On the opposite the odd peaks (those which appear for time delays which are odd multiple of 2 hours) start with very small amplitudes and grow quickly; later on they also show beatings.

For delays larger than 150 hours the amplitude of the envelope stops decreasing and shows random beating, but depends on the integration time, as shown on Fig. 2 and Fig. 3. To check if this part of the ACF is related to the existence of stable periodic components or to the statistical uncertainties, we study the changes of the ACF for partial integration over δ days: those ACF are oscillating functions, from which we compute the mean energy for delays between 200 h and 600 h. This function is itself

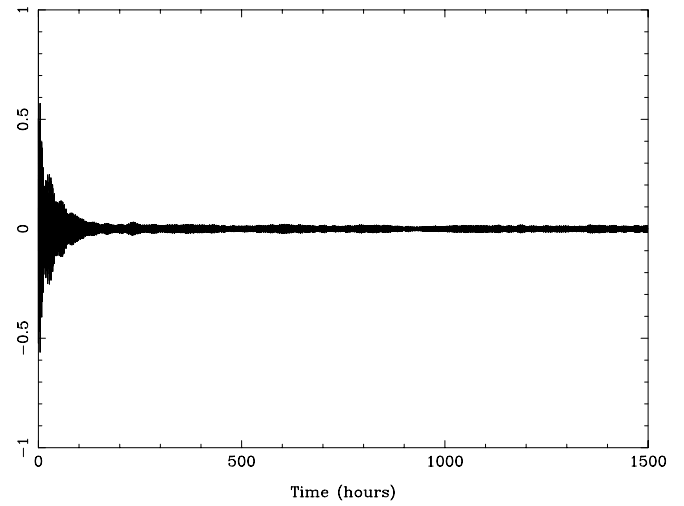


Fig. 2. The normalized ACF of the solar velocity for 550 days.

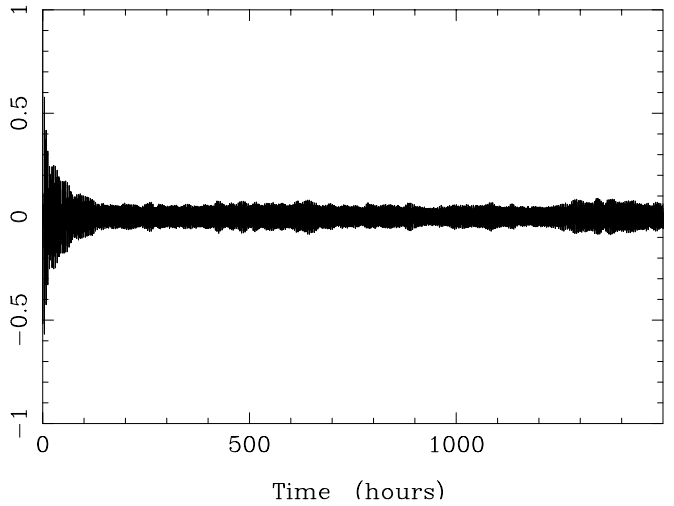


Fig. 3. The normalized ACF of the solar velocity for 50 days.

related to the sporadic behavior of the signal, so we compute it twice, starting from the beginning and from the end of the data set. The 2 sets of results are statistically independent up to $\delta = 250$ days, value related to T_o and to the maximum lag used in energy sum. The results are close to a law in $\sqrt{1/\delta}$ as expected for a function related to a stationary stochastic process (see Fig. 4.)

3. The ACF of the signal related to a single p-mode

If we assume that the signal is of infinite length and related to a stationary stochastic process, the ACF is the Fourier transform from the spectral density (DS). In the case of an observation interval T_o , this is an approximation valid if $\tau \ll T_o$. Using an FFT algorithm, we can thus extract in the DS the power related to one single solar mode, and compute the ACF of this single mode. We use a frequency table (Lazrek, 1998) slightly different from those already published (Lazrek *et al.*, 1997) and a data set of $T_o = 550$ days.

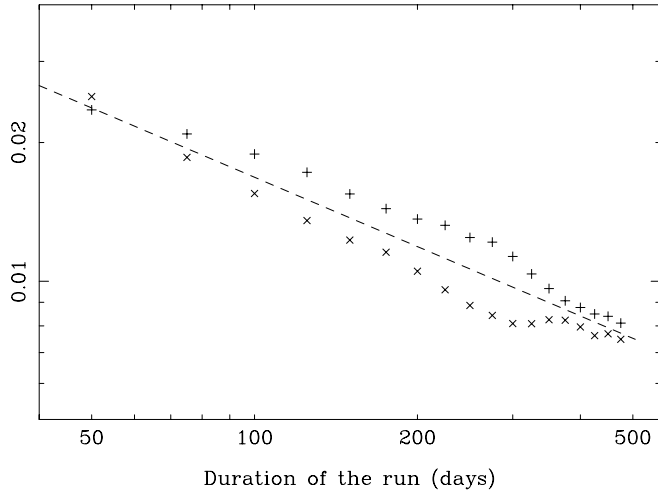


Fig. 4. Sum of the energy in the normalized ACF from 200h to 600h, vs. the duration δ of the integration time. The marks + are for the ACF computed with the first available data, \times for the ACF computed with the last data. The dashed line is a $\delta^{-1/2}$ function. Log-log scales.

This ACF depends on the statistical properties of the p-modes, as discussed in Sect. 7. For a Lorentz profile, the initial decrease follows an exponential law and the numerical model needs only 2 parameters: the damping time and the energy of the non-correlated noise. The energy related to this noise gives here a fringe centered at $T = 0$, with a shape depending on the filter's band-pass. In log vs. time scale, the fit is simply linear. The parameters of the numerical filter, central frequency and band-pass, have no direct influence on the ACF and can be used to improve the signal to noise ratio.

The damping time is related to the radial order. Figs. 5 and 6 show the normalized function $R_N(\tau)/R_N(0)$ for two radial modes, compared with an exponential law computed with the assumption of a damping times T_0 . The fringes of the ACF show that the statistical uncertainty is not yet negligible. The damping time is $T_0 = 90$ h. for the radial modes $\mathcal{N} = 18$ (an high energy mode) and increases to $T_0 = 120$ h. for the $\mathcal{N} = 14$ mode, as shown Fig. 5 and Fig. 6.

The damping times found in literature are deduced from the numerical adjustment of Lorentz's profile in the DS (Lazrek, 1997.) This technic give a half-linewidth W , related to the damping time by $W = 1/(2\pi T_0)$. The results of the 2 methods are in good agreement, but the analytic uncertainties are different, leading us to explore farther this new method. This preliminary study left aside the determination of the energy (vanishing in the normalization for $T = 0$) and of the p-mode frequency (as we don't study the oscillatory term of the ACF.) The statistical significance of those terms and their relation with the equivalent parameters coming from a maximum likelihood fit need also to be studied.

4. General discussion

Solar oscillations are assumed to be a stationary stochastic process with a finite average power. Then the power spectrum $P(\nu)$

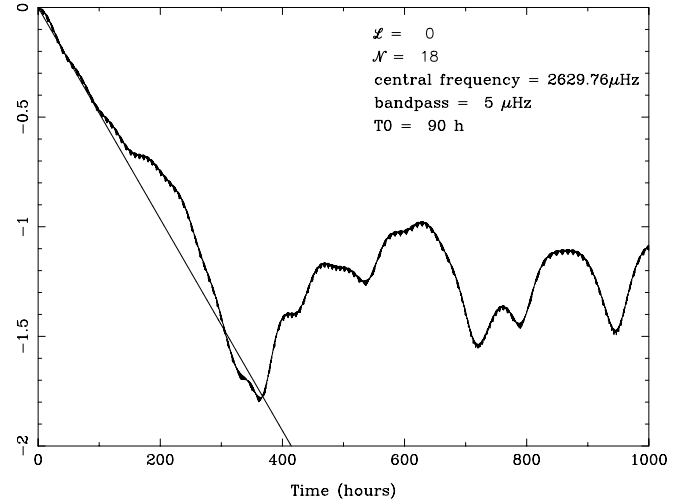


Fig. 5. The normalized ACF for the radial mode $\mathcal{N} = 18$, using a non-apodized $\pm 5\mu\text{Hz}$ filter. This function contains an oscillatory term, removed here for a better legibility. The fit of an exponential decrease $e^{-(t/T_0)}$ gives the damping time T_0 . (Log vertical scale, linear time scale.)

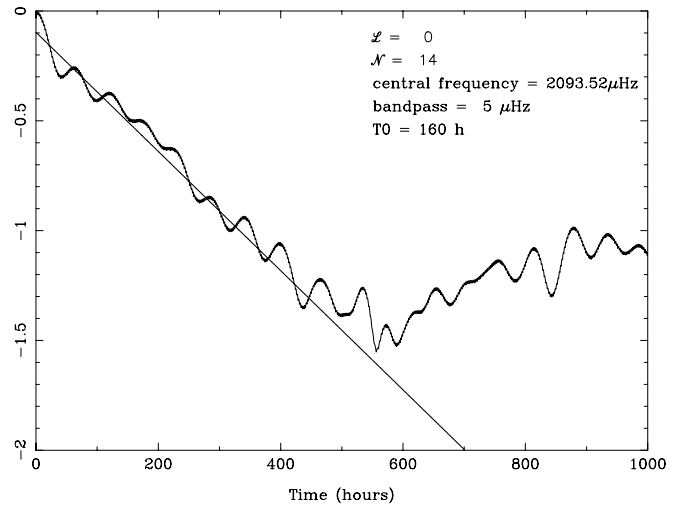


Fig. 6. Same as Fig. 5, but for the radial mode $\mathcal{N} = 14$. The energy in this mode is lower compared to the non-correlated noise, resulting in the fringe for small values of t .

is the Fourier transform of the statistical average of the correlation function $R(\tau) = E[x(t)x(t + \tau)]$ and conversely. Here and everywhere in the following $E[y]$ is the statistical average of y . If $R(\tau)$ has no periodic component then it is absolutely integrable as well as $P(\nu)$ and therefore from the Riemann-Lebesgue lemma we know that (Davenport & Root 1958, pg 107)

$$\lim_{\tau \rightarrow \infty} R(\tau) = 0 \quad (1)$$

$$\lim_{\omega \rightarrow \infty} P(\omega) = 0 \quad (2)$$

When Eq. (1) is not fulfilled there is a periodic component in the signal of the form $\sum_j A_j \cos(\omega_j t + \phi_j)$ where the A_j and the ϕ_j are deterministic quantities.

However, we only know $R(\tau, T) = R_N(\tau)$ for T smaller or equal to half the observation time T_o and though it is quite long, it remains finite. This forbid us to tell off-hand whether eq (1) is satisfied or whether $R(\tau)$ remains finite at infinity without further informations. Therefore it is necessary to investigate at which rate the correlation function goes to zero for a stochastic signal.

Let a_j and b_j ($j > 0$) be the coefficients of the cos and sin Fourier series over the time interval T . We define the power (per Hz) as $P_j = T(a_j^2 + b_j^2)/2$ and we also have $E[P_j] = TE[a_j^2]$ (see Gabriel 1993b). Since (see Gabriel 1993b)

$$E[P_j] = \int_0^\infty E[P(\omega)]f(\omega - \omega_j)d\nu$$

with

$$f(\omega - \omega_j) = \frac{\sin[(\omega - \omega_j)T/2]}{(\omega - \omega_j)T/2}$$

$E[P_j]$ is nearly time independent for long enough observation times since $\lim_{T \rightarrow \infty} E[P(\nu, T) - P(\nu)] = 0$. It can be obtained either observationally (see for instance Gabriel 1994, 1995) or theoretically (Gabriel 1993a). It is known that the p.d.f. of the power at a Fourier point $p(P_j)$ is given by a χ^2_2 (see for instance Gabriel 1993b)

$$p(P_j) = \frac{1}{E[P_j]} \exp\left[-\frac{P_j}{E[P_j]}\right]$$

and since $E[P_j]$ is nearly time independent for long enough observation times, a periodogram will never converge towards the power spectrum whatever the observation length but will always show fluctuations around the mean profile. This is not a new result indeed but a consequence of the theorem which states that the periodogram of a function which is not absolutely integrable does not converge in the mean towards the power spectrum for an infinite observation time i.e.

$$\lim_{T \rightarrow \infty} E[(P(\nu, T) - P(\nu))^2] \neq 0$$

when $P(\nu) \neq 0$ (see for instance Davenport & Root 1958, pg. 108). However it has important consequences on the behavior of the correlation function. Let us split P_j in the following way:

$$P_j = E[P_j] + \{P_j - E[P_j]\}$$

The first term of the right hand side member is deterministic and when j varies it draws the mean line profiles. The second term which is stochastic will appear in the spectrum as a succession of delta functions which looks like the spectrum of a stationary non stochastic periodic signal.

Since

$$x(t) = \sum_{j=-N/2}^{N/2} c_j \exp[i\omega_j t]$$

where N is the total number of observations, $c_j = (a_j + ib_j)/2$ ($j > 0$) and $c_{-j} = c_j^*$, then

$$R(\tau, T) = \sum_{j>0} 2|c_j|^2 \cos(\omega_j \tau) = \frac{1}{T} \sum_{j>0} P_j \cos(\omega_j \tau)$$

or

$$R(\tau, T) = \frac{1}{T} \sum_{j>0} E[P_j] \cos(\omega_j \tau) + \frac{1}{T} \sum_{j>0} \{P_j - E[P_j]\} \cos(\omega_j \tau) \quad (3)$$

The first term of this equation is deterministic and will give the correlation function of the mean line profiles (just replace the sum by an integral). For instance, for Lorentzian lines, it will give a sum of terms each of them approaching $\cos(\omega_c \tau) \exp(-\Gamma \tau)$ (ω_c is the central line frequency and Γ is the damping time) for large enough observation times. This term will always go to zero as τ goes to infinity. The second term is the stochastic one and it has a zero mean. Since the variance of each P_j is nearly time independent, the variance of this term is proportional to the number of terms in the sum (and therefore to the observation time T) divided by T^2 . It follows that the standard deviation of that term will vary as $\sqrt{1/T}$. This is exactly the way observations vary for large τ values when $\Gamma T \gg 1$. This second term of Eq. (3) is a sum of trigonometric functions which correspond to the interpretation given above of the related term in the periodogram.

Since the variance of the stochastic term in Eq. (3) goes slowly to zero it is possible that it still contributes significantly to the correlation function after 550 days but because, for large time lags, the amplitude of the ACF decreases as expected for a stochastic signal there is no need to assume that the solar signal contains a deterministic component.

5. The model of randomly excited oscillators

Because the time autocorrelation function which was computed from the observations does not assume that the observed signal has a period T and also because it is difficult to proceed without some assumption concerning the line profiles, we have to continue the discussion with a simplified model. We assume that the observed signal is the sum of stochastically excited damped oscillators with frequencies ω_n and damping times Γ_n . The only approximation behind this model is that it assumes that all lines have a Lorentzian profile. The signal is then given by:

$$x(t) = \sum_n \sum_{t_j^n=-\infty}^{\infty} A_n(t_j^n) H(t - t_j^n) \sin[p_n(t - t_j^n)] \exp[-\Gamma_n(t - t_j^n)] \quad (4)$$

where $p_n = \sqrt{(\omega_n^2 - \Gamma_n^2)}$, t_j^n is an excitation time and H is the Heaviside function.

Let T_o be the observation time, the time autocorrelation function is defined by:

$$y(\tau) = \frac{1}{T} \int_0^T x(t)x(t+\tau)dt \\ = \sum_k x(t_k)x(t_k+\tau)W(T-t_k)$$

where $T + Max(\tau) = T_o$ but the integration time T may be much smaller than T_o and $W(T-t_k)$ equal 1 if $(T-t_k) > 0$ and zero otherwise.

Taking Eq. (4) into account, we find :

$$y(\tau) = \sum_n \sum_{t_i^n=-\infty}^T \sum_{t_j^n=-\infty}^{T+\tau} \frac{A_n(t_i^n)A_n(t_j^n)}{4\Gamma_n T} \\ \cos[p_n(\tau + t_i^n - t_j^n)]\mathcal{I}(t_i^n, t_j^n, t_*^n) \quad (5)$$

with $t_*^n = Max(t_i^n, t_j^n - \tau, 0)$ and

$$\mathcal{I} = \exp[-\Gamma_n(t_i^n + t_j^n - \tau - 2t_*^n)]\{1 - \exp[-2\Gamma_n(T - t_*^n)]\}$$

To get this expression we have assumed that all differences between the p_n are large compared to Γ_n to avoid any significant coupling between modes. This implies that we have neglected the rotational splitting.

The ACF given in fig.1 and 2 are just the special case for $T = T_o/2$. In practice, we have computed correlation functions normalized for $\tau = 0$.

Since excitations are considered to take place close to the surface of the convection zone in connection with the granule dynamics, the number of excitations is expected to become quickly very large. This suggests to split Eq. (5) in two parts. The first one, obtained for $t_i^n = t_j^n$, is given by:

$$y_1(t) = \sum_n \sum_{t_i^n=-\infty}^T \frac{A_n^2(t_i^n)}{4\Gamma_n T} \cos(p_n \tau)\mathcal{I}(t_i^n, t_i^n, t_*^n) \quad (6)$$

With a good approximation when $\Gamma_n T \gg 1$, it leads to:

$$y_1(t) = \sum_n \frac{[A_n^2(t_i^n)]}{4\Gamma_n T_{ex}^n} \cos(p_n \tau) \exp[-\Gamma_n \tau] \quad (7)$$

where T_{ex}^n is the average time between two excitations for line “ n ” and $[A_n^2(t_i^n)]$ is the average of $A_n^2(t_i^n)$ over the time T . This first part of the ACF is deterministic and corresponds in the frequency domain to smooth Lorentzian lines.

The second part gives:

$$y_2(\tau) = \sum_n \frac{[A_n^2(t_i^n)]}{4\Gamma_n T} \sum_{t_i^n=-\infty}^T \sum_{t_j^n \neq t_i^n}^{T+\tau} \cos[p_n(\tau + t_i^n - t_j^n)] \\ \mathcal{I}(t_i^n, t_j^n, t_*^n) \quad (8)$$

or

$$y_2(\tau) = \sum_n \frac{[A_n^2(t_i^n)]}{4\Gamma_n T_{ex}^n} C_n \cos[p_n \tau + \phi_n] \quad (9)$$

with

$$C_n = \sqrt{(C_c^n)^2 + (C_s^n)^2}$$

$$C_c^n = \frac{T_{ex}^n}{T} \sum_{t_i^n=-\infty}^T \sum_{t_j^n \neq t_i^n}^{T+\tau} \frac{A_n(t_i^n)A_n(t_j^n)}{[A_n^2(t_i^n)]} \cos[p_n(t_i^n - t_j^n)] \\ \mathcal{I}(t_i^n, t_j^n, t_*^n)$$

$$C_s^n = \frac{T_{ex}^n}{T} \sum_{t_i^n=-\infty}^T \sum_{t_j^n \neq t_i^n}^{T+\tau} \frac{A_n(t_i^n)A_n(t_j^n)}{[A_n^2(t_i^n)]} \sin[p_n(t_i^n - t_j^n)] \\ \mathcal{I}(t_i^n, t_j^n, t_*^n)$$

$$\tan \phi_n = \frac{C_s^n}{C_c^n}$$

The C_n give the the magnitude of the stochastic term relative to the deterministic one for mode n .

This second part of the ACF is stochastic and has a zero mean. The ϕ_n are uniformly distributed in $[0, 2\pi]$ and, given the large number of excitations, we can assume that the C_c^n and C_s^n are Gaussian. Their variance can easily be obtained and with our assumption that $\Gamma_n T \gg 1$ it is independent of τ . We get:

$$\sigma^2(C_c^n) = \sigma^2(C_s^n) = \frac{1}{2\Gamma_n T}$$

As expected, we find again that the variance varies as T^{-1} . It is interesting to notice that for $\Gamma_n T \gg 1$ the correlation function (and also the power spectrum) is function of only two physical parameters per line: the average energy received per second $[A_n^2(t_i^n)]/T_{ex}^n$ and the damping time Γ_n^{-1} . Therefore, there is no way to distinguish between weak but frequent excitations and strong but sparse ones as long as there many of them during one damping time.

The p.d.f of C_n^2 is given by:

$$p(C_n^2)dC_n^2 = \Gamma_n T \exp[-(C_n^2)\Gamma_n T]dC_n^2 \quad (10)$$

It remains to check whether it is possible to find a set of values for the C_n and ϕ_n which reproduce the main behaviors of the observations and which have an acceptable probability to be realized.

Before discussing the results of numerical simulations, one point needs to be discussed to help understanding why, for small times, the observed ACF decreases with a characteristic time much shorter than the damping time of the strongest lines.

Let us see what would be the expression for $y_2(\tau)$ (and also that of $y_1(\tau)$) for $\Gamma_n \tau \ll 1$, if the mode spacing was constant, i.e. if the first order asymptotic theory was accurate. We would have:

$$p_n = \omega_0 + n\Delta\omega$$

with $\Delta\omega/(2\pi) \simeq 67 \mu\text{Hz}$ and we would obtain from Eq. (9):

$$y_2(\tau) = \sum_n \frac{[A_n^2(t_i^n)]}{4\Gamma_n T_{ex}^n} C_n \{ \cos[n\Delta\omega\tau] \cos[\omega_0\tau + \phi_n] \\ - \sin[n\Delta\omega\tau] \sin[\omega_0\tau + \phi_n] \}$$

Under that hypothesis, the coefficients of $\cos[\omega_0\tau + \phi_n]$ and $\sin[\omega_0\tau + \phi_n]$ are periodic functions with a period of about 4 hours and a complex peak structure such as shown by the observations cannot be reproduced. Each time that $\Delta\omega\tau = 2k\pi$, i.e. when τ is a multiple of about 4 hours, the amplitude of $y_2(\tau)$ is maximum and nearly constant since $\omega_0 \gg \Delta\omega$. At these times the $l=0$ and 2 pairs interfere constructively with the $l=1$ and 3 pairs. On the other hand, each time that $\Delta\omega\tau = (2k+1)\pi$, then $\cos[n\Delta\omega\tau]$ alternates between $+1$ and -1 and $y_2(\tau)$ is small. Then the even and odd degrees pairs and interfere destructively. Notice that the terms in $\sin[n\Delta\omega\tau]$ will never have a large contribution for $\Delta\omega\tau = (k+1/2)\pi$.

In the case where the coefficients of the curled braces are constant, if $N+1$ is the number of lines, we obtain:

$$\sum_{n=0}^N \cos[n\Delta\omega\tau] = \frac{\sin[(N+1/2)\Delta\omega\tau]}{2\sin[\Delta\omega\tau/2]} + \frac{1}{2}$$

$$\sum_{n=0}^N \sin[n\Delta\omega\tau] = \frac{\sin[(N+1)\Delta\omega\tau/2] \sin[N\Delta\omega\tau/2]}{\sin[\Delta\omega\tau/2]}$$

For $\Delta\omega\tau = 2k\pi$, the first equation is equal to $(N+1)$ while the second one is always zero. For $\Delta\omega\tau = (2k+1)\pi$, the first equation is equal to 1 for even N and 0 for odd ones and the second one is again equal to zero. The first equation is strongly peaked at $\Delta\omega\tau = 2k\pi$ while the second one has smaller maxima for $\Delta\omega\tau$ a few degrees larger (depending upon the value of N).

This behavior is completely different from that of the observed ACF. Therefore the fast initial decrease of the peak amplitudes at $\tau \simeq k * 4$ hours and their later modulation as well as the initial increase and the later modulation of the peaks at $\tau \simeq (2k+1) * 2$ hours can only be produced if we take into account the non equi-spacing of the modes which destroys the regularity of the interference pattern between even and odd degree pairs.

6. Results of numerical simulations

The sum of $y_1(t)$ and $y_2(t)$ has been computed and compared with observations. For this, the frequencies have been taken from GOLF and the line widths from Chaplin et al. (1997), Fierry-Fraillon et al. (1997a) and from Fierry-Fraillon et al. (1997b). The three sets of line widths though slightly different lead to no visible difference in the ACF because as stated above the variations of the peak amplitudes for not too large delay times are mainly controlled by the mode spacing and not by the damping times. The other parameters appearing in Eqs. (7) and (9) are related to the excitations and are of statistical type. From the observations we expect that they vary with the integration time T . We think that the aim of a numerical simulation is to reproduce the main characteristics of the observations with a model as simple as possible also we have decided to choose the values of these parameters in a way as simple as possible. First, we must decide of the frequency dependence of the mode power which is proportional to $G(\nu)/(\Gamma_n M_n)$ where $G(\nu)$ is the acoustic power provided to one mode by convection and M_n

Table 1. Visibility factors for the power normalized to one for the radial modes.

l	No limb darkening	Eddington	$\cos(0.66\theta)$
1	27/16	1.80866	1.7778
2	0.8	1.02578	0.96552
3	7/64	0.2264	0.19167
4	0.	0.02737	0.0051478
5	0.03125	0.02914	0.001141

is the mass of the mode. The M_n are taken from the theoretical adiabatic eigenvalues. For the $G(\nu)$, we have tried 2 slightly different forms according to the prescription of Libbrecht (1988) and Osaki (1990).

- proportional to ν^8 below ν_{crit} μHz and proportional to $\nu^{-5.5}$ above that value
- proportional to ν^8 below ν_{crit} μHz , constant between ν_{crit} and 3500 μHz and proportional to $\nu^{-5.5}$ above that value

These 2 choices as well as small variations of the exponents do not lead to significant modifications in the ACF. The value $\nu_{crit} = 3300$ μHz provides the best fits.

There are variations of the mode powers around the mean values discussed above indeed but we do not allow variations from mode to mode but only for all the modes of a given degree. We have include these variations into the visibility factors.

Secondly the amplitude of the ACF is also a function of the visibility factor for each degree. We have used as a guide the values of the visibility factors without limb darkening, with the limb darkening function given by the Eddington approximation and the expression used by Robillot (1997) in Bordeaux for observations with the MR5 instrument (it is given by $\cos(0.66\theta)$). The values obtained assuming that the power is the same for all degrees are given in Table 1. It shows that as the degree increases the visibility factor becomes more and more sensitive to the choice of the limb darkening function. Since they are small the chosen values for $l=4$ and 5 are unimportant.

Finally, we have used the same value of C_n and ϕ_n for all the modes though they are also expected to vary from mode to mode. The assumption of one unique value for ϕ is without effect because the oscillation periods are much shorter than two hours period of the resonances.

The solar signal has also energy outside the 5 min. domain. The fraction of energy in the identified modes of degrees zero to three has been estimated observationally to 2/3. To take this into account, we have added a white noise to the signal given by Eqs. (7) and (9). The energy of that white noise has been set to 1/2.9 of the total energy. This allow to obtain the right amplitudes for the first two peaks.

We have first generated an artificial set of frequencies verifying the first order asymptotic theory. The results obtained assuming that the oscillations are not damped or using the same damping times as for the real data are shown in Figs. 7 and 8 respectively. They show only large resonances with a period of about 4 hours and the peaks at odd multiples of two hours are very small. This corroborates the simple discussion of the

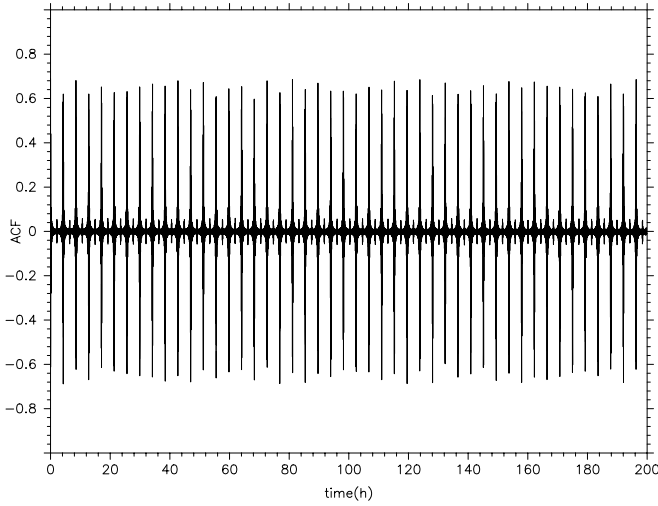


Fig. 7. ACF for equi-spaced and undamped frequencies

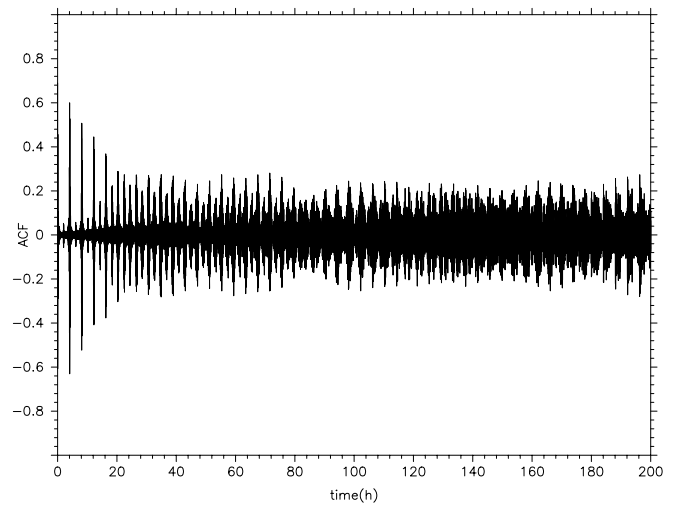


Fig. 9. ACF for observed but undamped frequencies

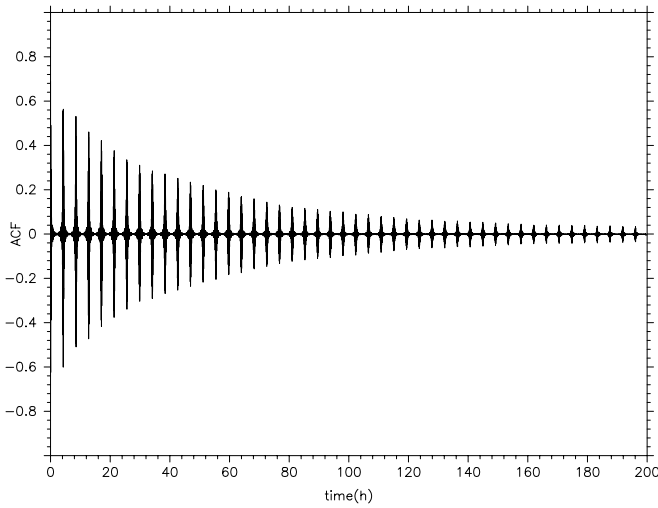


Fig. 8. ACF for equi-spaced frequencies but with the observed line widths

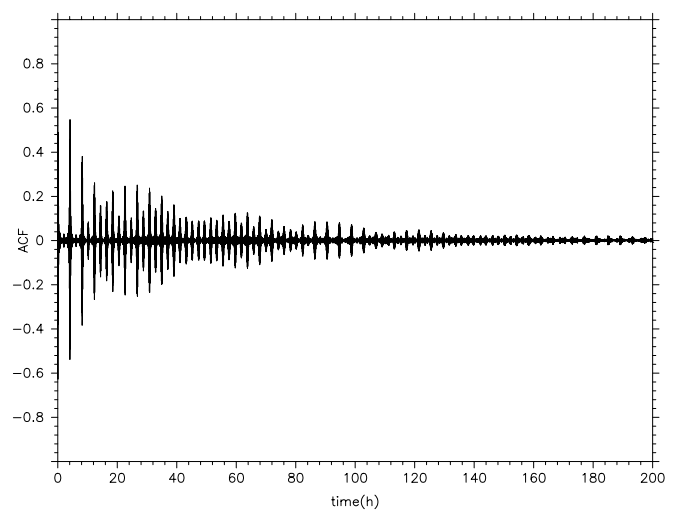


Fig. 10. ACF for observed damped frequencies with the visibility factors equal to 1, 1.8, 1 and 0.19 for $l=0$ to 3 respectively and $C=0.05$

previous section. Figs. 9 and 10 have been obtained with the same set of parameters but with the observed frequencies. The differences are striking. Without damping the modulation of the amplitudes is much larger and the peaks at odd multiples of two hours are much larger (except for small times). For large times, the amplitudes are too large indeed but for times smaller than about 40 hours it already shows several similarities with the observed ACF given in Fig. 1, mainly the variation with the delay time of the peak amplitudes. With damping, we obtain an ACF very similar to the observed one.

From the visibility factor, we expect about the same amount of energy in the $l = 0$ and $l = 2$ modes. Fig. 10 gives the ACF with the visibility factors for degrees zero to three equal respectively to 1, 1.8, 1 and 0.19 and $C = 0.05$. The power of the even peaks reaches too low values close to 30 hours and the ratio of the even and odd peak amplitudes is not right close to 50 hours. To get a solution closer to the observations for an observation time equal to 550 days, we have to multiply the

power in the $l = 2$ modes by a factor 0.6 in order to have the right behavior around 30 to 50 hours and also to reduce the power in the odd degrees in order to keep small odd peak amplitudes for small times. Taking for the visibility factors of the $l=0$ to 3 modes respectively 1, 1.5, 0.6 and 0.15, we obtain an ACF, given in Fig. 11, very similar to the observed one.

Fig. 12, computed with the same parameters as Fig. 11 but with $\nu_{crit} = 3000 \mu\text{Hz}$, shows the influence of that parameter. It is seen that the agreement is not as good but the degradation of this fit is not big.

Fig. 13 is computed with the same parameters as Fig. 11 but gives the ACF for times up to 1500 hours. It shows that for times larger than about 200 hours the amplitudes are nearly constant with modulations due to mode beatings.

The main difference between the observed ACF for observation times of 550 and 50 days is the amplitudes for large times. To increase it, we have to take a larger value for C . Taking

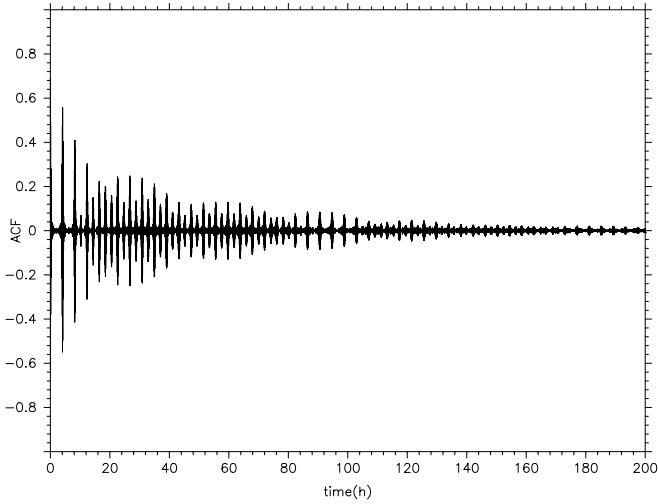


Fig. 11. ACF for observed damped frequencies with the visibility factors equal to 1, 1.5, 0.6 and 0.15 for $l=0$ to 3 respectively and $C=0.05$

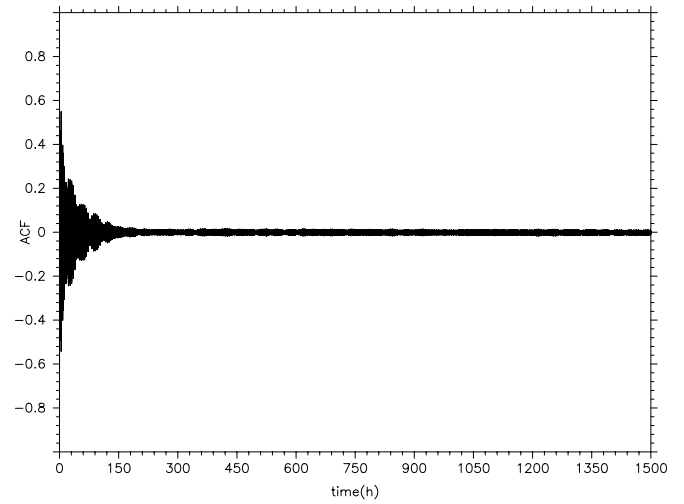


Fig. 13. Same as Fig. 11 but for time lags extending up to 1500 hours

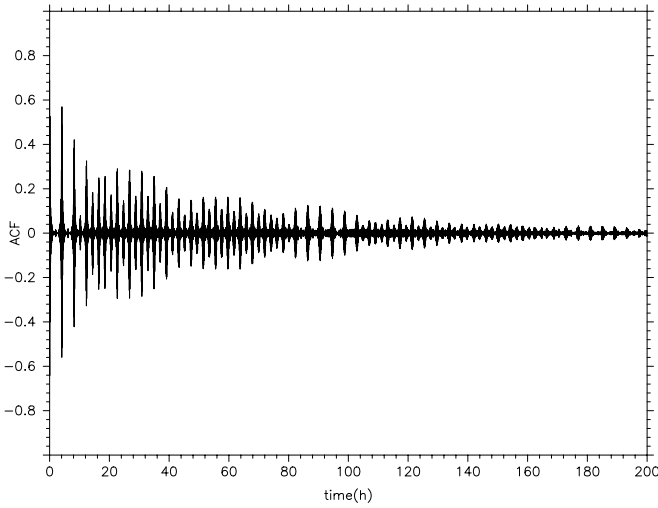


Fig. 12. Same as Fig. 11 but with $\nu_{crit} = 3000 \mu\text{Hz}$

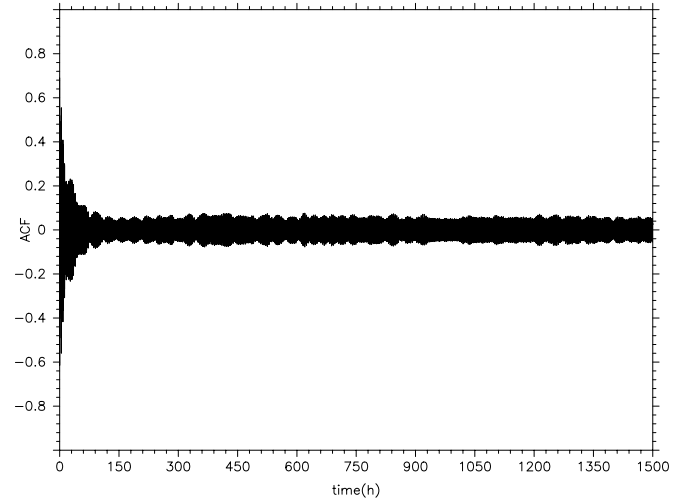


Fig. 14. Same as Fig. 13 but with $C=0.2$

$C = 0.2$, we get the fig 14 which has about the right amplitudes for large times. It has to be compared to Fig. 3. The other difference with Fig. 13 is that the amplitudes stop decreasing earlier just because y_2 as been increased and one gets earlier $y_1 < y_2$.

As time grows, we expect that the approximations made in order to keep a simple model (no rotational splitting, no amplitude variation around the mean from mode to mode and only one value for the C_n) influence the mode interferences and indeed the peaks of the amplitude modulation do no longer occur at the right times. Also we cannot exclude that if one or several of the above approximations were relaxed a good fit could be obtained with another set of parameters. Nevertheless it is remarkable that the main characteristics of the ACF are reproduce with the simple model.

Since the amplitudes are the largest for modes with full line widths of the order to one to two μHz , we have $\Gamma T \simeq 224$ for $T_o = 550$ days and taking an average line width of 1.5 μHz .

Taking Eq. (10) into account, we see that the probability to have a value of C larger than 0.02 is of the order of 0.91 which makes the adopted value of C acceptable. Similarly for $T_o = 50$ days the probability to have $C > 0.2$ is 0.44. Therefore the observations can be explained by the model of stochastic oscillations without the necessity to call upon periodic components in the signal.

7. Application to the p-mode study

The discussion in Sect. 4 suggests a method to study the statistical properties of the p-modes. If we equal the power spectrum to zero everywhere but in the frequency domain of one line and then take the Fourier transform of that spectrum, we isolate one term of the sum over j in Eq. (3). Since the stochastic term becomes smaller and smaller compared to the deterministic one as the observation time increases, we may hope that the GOLF ACF will converge to the ACF of the p-mode line profile.

For a Lorentz profile, the deterministic term is proportional to $\cos(\omega_c t) * \exp(-\Gamma t)$ and it is possible to obtain the value of the e-folding time using the ACF for the short time lags ($T < 10$ to 25 days, see Sect. 3). It is possible to refine the method to study the line profiles and to extend it to non-radial modes. After choosing an analytic expression for the line profile containing several parameters (the maximum power, the peak frequency, the line width and one or several asymmetry coefficients and also one or two noise parameters), one can compute the corresponding ACF and obtain the parameters through a fitting with the observed ACF for not too long time delays. For non-radial modes, several lines have to be treated together and the rotational splitting provides one extra parameter.

Though the preliminary study reported in Sect. 3 shows that line widths can be obtained without difficulty, it is well known that informations concerning line asymmetry are difficult to obtain for low degree modes and we cannot tell off hand what the full set of reliable parameters provided by this method will be. This general method requires a more thorough fitting method than used here and it is presently under development.

8. Conclusion

1. The variation of the amplitude of the even peaks at the beginning of the ACF is not related to the damping time but is a consequence of the non constancy of the frequency separation between modes (or of the deviations relative to the first order asymptotic approximation). Except for a slow exponential decay, which is due to the damping of the modes, these deviations also explain the time variations of the peak amplitudes of even and odd harmonics.
2. For large delay times, the standard deviation of the ACF of stochastically excited oscillations varies as the reciprocal of the square root of the integration time, exactly as found in the observations. This provides one more argument in favor of the stochastic excitation of solar p-modes.
3. The behavior at large times can be explained with a very simple model which assumes that C is of the order of 0.2 and 0.05 for observation times of 50 and 550 days respectively. This apparently deterministic signal is, in fact, produced by the slow decrease of the standard deviation of stochastically excited oscillations ACF.
4. A new method to study line profiles which has analytic uncertainties different from these using the power spectrum, has been proposed. First tests on radial modes show that it can at least provide reliable line widths. The general method is presently under development.

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References

- Chaplin, W.J., Elsworth, Y., Isaak, G.R., McLeod, C.P., Miller, B.A. & New, R., 1997, MNRAS, 288, 623
- Davenport, W.B.Jr. & Root, W.L., 1958, An introduction to the theory of random signal and noise, Mc Graw-Hill
- Ehgamberdiev, S., Khalikov, S., Lazrek, M., & Fossat, E., 1992, A&A, 253, 252
- Fierry Fraillon, D., Gelly, B., Schmider, F.X., Leibacher, J.W., Hill, F., Fossat, E. & Pantel, A., 1997a, A&A (in press)
- Fierry Fraillon, D., 1997, private communication
- Gabriel, A.H., Charra, J., Grec, G., Robillot, J.M., Roca Cortès, T., Turck-Chièze, S., Ulrich, R.K., Basu, S., Baudin, F., Bertello, L., Boumier, P., Charra, M., Christensen-Dalsgaard, J., Decaudin, M., Dzitko, H., Foglizzo, T., Fossat, E., Garcia, R.A., Herreros, J.M., Lazrek, M., Pallé, P.L., Pétrou, N., Régulo, C., Renaud C.: 1997, *Solar Phys.* 175, p 207-226
- Gabriel, M., 1993a, A&A, 274, 935
- Gabriel, M., 1993b, A&A, 274, 931
- Gabriel, M., 1994, A&A, 287, 685
- Lazrek, M., Baudin, F., Bertello, L., Boumier, P., Charra, J., Fierry-Fraillon, D., Fossat, E., Gabriel, A.H., Garcia, R.A., Gelly, B., Gouiffes, C., Grec, G., Pallé, P.L., Pérez Hernández, F., Régulo, C., Renaud, C., Robillot, J.M., Roca Cortés, T., Turck-Chièze, S., Ulrich, R.K. : 1997, *Solar Phys.* 175, p227-246
- Lazrek, M.. 1998, private communication.
- Libbrecht, K.G., 1988, ApJ, 334, 510
- Osaki, y., 1990, Progress of Seismology of the Sun and Stars, ed. Y. Osaki & H. Shibahashi, Springer-Verlag, pg. 75.
- Robillot, J.M., 1997 private communication.