The IRAF/NOAO Spectral World Coordinate Systems

Francisco Valdes
IRAF Group, NOAO[1], PO Box 26732, Tucson, AZ 85726

Abstract.
The world coordinate systems for dispersion calibrated spectra used in the IRAF/NOAO spectroscopy packages are presented. In particular, the image header keywords which define the coordinates. These keywords appear both as part of the IRAF image structure and map directly to FITS format. The types of spectra include multidimensional images with one or more spatial axes and a linear or log-linear dispersion axis and special *equispec* and *multispec* formats having multiple independent one dimensional spectra in a single multidimensional image. The *multispec* format also includes general nonlinear dispersion coordinate systems using polynomial, spline, sampled table, and look-up table functions.

1. Introduction

IRAF[2] version 2.10 is the first release to provide general support for user world coordinate systems (WCS) in image and photon list data. For spectra the world coordinate systems include at least one dispersion coordinate; i.e wavelength, frequency, etc. The NOAO spectroscopic packages were extensively modified to make use of the new coordinate system capabilities and to add non-linear dispersion coordinate systems to the previously available linear and log-linear systems.

Within the IRAF environment users need not concern themselves with the details of the WCS. However, to export the spectral coordinate systems to other software or to import dispersion calibrated spectra into IRAF requires understanding the external representation of the spectral coordinate systems. By this we mean a FITS image header. Because the current IRAF image formats use the same syntax as FITS to store image header information, such as the WCS, this description also applies to IRAF spectral images. The purpose of this paper is to describe the spectral WCS FITS representations in sufficient detail to allow someone to interpret them and possibly implement software outside of IRAF to access them. This description applies to version 2.10.3 though much of it also applies to earlier versions of 2.10.

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[2] Image Reduction and Analysis Facility, distributed by the National Optical Astronomy Observatories

2. Types of Spectral Data

The spectra described in this paper are stored as images. Images may be one, two, or three dimensional with one axis being the dispersion axis. Generally a pixel value is then the flux over some interval of wavelength and position. The simplest example of a spectrum is a one dimensional image where the pixel value is the sum over some spatial extent.

There are two types of higher dimensional spectral image formats. One type has spatial axes for the other dimensions and the dispersion axis may be along any of the image axes. Typically this type of format is used for long slit (two dimensional) and Fabry-Perot (three dimensional) spectra. This format will be referred to in this paper as *spatial* format.

The second type of spectral image format consists of multiple one dimensional spectra stored in a higher dimensional image with the first image axis being the dispersion axis. This format allows associating many spectra and related parameters into a single data object. This format is called *multispec* format. A special case of this is when all spectra have the same linear dispersion relation in which case a simpler WCS representation is used and the format is called *equispec*. These formats are important since maintaining large numbers of one dimensional spectra as individual images is very unweildy for the user and inefficient for the software.

Examples of equispec/multispec format are the extracted spectra from a multifiber or multiaperture spectrograph or the extracted orders from an echelle spectrum. The second axis is some arbitrary indexing of the spectra, called *apertures* in IRAF tasks, and the third dimension is used for associated quantities. The IRAF **apextract** package may produce multiple spectra from a CCD image in successive image lines with an optimally weighted spectrum, a simple aperture sum spectrum, a background spectrum, and sigma spectrum as the associated quantities along the third dimension of the image.

Many IRAF tasks which are designed to operate on one dimensional spectra may operate on spatial spectra by internally summing a number of neighboring spectra across the dispersion axis. This eliminates the need to extract one dimensional spectra from the natural format of this type of data in order to use tasks oriented towards the display and analysis of one dimensional spectra.

One dimensional spectra, whether from a multispec/equispec image or implicitly defined from spatial spectra, have several associated quantities which may appear in the image header as part of the coordinate system description. The primary identification of a spectrum is an integer aperture number. This number must be unique within within a single image. There is also an integer beam number. This is used for various purposes. Two common uses are to discriminate object, sky, and arc spectra in multifiber/multiaperture data and to identify the order number in echelle data. For spectra implicitly defined from spatial data the aperture and beam numbers identify the the spatial axes (for example the line and band when the dispersion is along the column axis). Since most 1D spectra are derived from an integration over one or more spatial axes two additional aperture parameters recorded are the aperture limits. These limits refer to the original pixel limits along the first spatial axis (there is no generalization to two spatial axes). This is primarily for record keeping but in some cases it is used for spatial interpolation during dispersion calibration.

These values are currently set either by the **apextract** tasks or when summing neighboring vectors in spatial spectra.

One additional WCS parameter which appears in the description below is a doppler factor. The equispec WCS records this value and also folds it into the WCS coefficients. In contrast, the multispec WCS does not modify the WCS coefficients but applies it separately whenever a wavelength is evaluated. The spatial format WCS does not include a doppler factor.

3. IRAF Mini-World Coordinate System

The IRAF/NOAO spectral world coordinates are part of the IRAF world coordinate system. This is an IRAF system interface used by applications to get and set coordinates associated with images and other data structures (the software extends beyond just *images* to such things as photon event lists). This allows system routines, such as those dealing with image sections, to modify the WCS transparently to the applications and the applications to access various coordinate systems without knowing how they are implemented. In particular, IRAF applications outside of the spectral packages can access the spectral world coordinates. The system interface is called the *IRAF Mini-World Coordinate System*, or MWCS, and it is discussed in detail by Tody (1990). In order to describe the spectral world coordinates it is necessary to understand a couple of basic concepts in the MWCS.

For applications that simply read coordinates, such as a display task, it is a simple matter to hide the details of the coordinate system. However, for applications that modify the image array sampling, and thus the relation between pixel coordinates and world coordinates, this is more difficult. For a large class of general image processing applications the operations modify the sampling in a simple linear fashion. Examples of such operations are subsampling, block averaging, block replicating, linear resampling, transposing, and rotating. The approach adopted by the MWCS is to separate the mapping from pixel coordinates to world coordinate coordinates into two steps; a linear transformation to the original pixel coordinate system upon which the world coordinate system was initially defined and then a transformation to the world coordinate system. Thus, the image operators need only define the linear transformation needed to restore the original pixels coordinates.

There is an additional complexity in that some image operations alter the dimensionality of the image. When the dimensionality is increased the applications generally add a new linear pixel coordinate system for the new axes. The more common case of dimensional reduction, such as extracting a piece of the image with an image section, is handled by the MWCS by maintaining the WCS description in the original dimensionality and adding a keyword describing the missing axes.

The MWCS defines three coordinate systems. The pixel coordinates of the current image array are called the *logical* coordinates. The original coordinates upon which the world coordinates were defined are called the *physical* coordinates. And, of course, the last set of coordinates are the *world* coordinates. In many cases two or three of these coordinates are the same; lacking a world coordinate system the world coordinates are the physical coordinates and if no resampling operation has been performed the physical and logical coordinates are the same. Note that the physical coordinates may also be of interest to a user.

The external representation of the logical to physical transformation is via the keywords LTVi and LTMi_j where i and j are axis numbers. The LTV keywords define an origin vector and the LTM keywords define a scale/rotation matrix. By convention missing values default to zero. The transformation is then given by

$$\vec{l} = |m| * \vec{p} + \vec{v} \tag{1}$$

where $\vec{l}$ is the logical coordinate vector, $\vec{p}$ is the physical coordinate vector, $\vec{v}$ is the translation vector represented by the LTV keywords and $|m|$ is the scale/rotation matrix. For the case of no rotation the coordinates of the first axis would be given by

$$l_1 = \mathtt{LTM1\_1} * p_1 + \mathtt{LTV1}$$

Internally the mapping from logical to physical to world coordinates applies to all of the world coordinate types. However, to conform to the FITS definition which maps logical pixel coordinates directly to world coordinates, those MWCS coordinate systems which can be represented by the FITS keywords CRPIXi, CRVALi, CDELTi are stored with the linear transform folded in. This is done by adjusting CRPIX and CDELT/CD. In keeping with this approach, any doppler factor is also folded into CRVAL and CDELT/CD.

The current FITS convention only defines a linear transformation from logical to world coordinates based on the above keywords. A proposal is under development (Hanisch and Wells 1992) that includes many sky projections based on the same keywords as well as defining a more general scale/rotation matrix using the keywords CDi_j. The type of sky projection is specified by the CTYPE keywords. The IRAF WCS currently implements these and uses the CDi_j convention for the scale/rotation matrix.

The reason for this discussion about the three MWCS coordinate systems and the FITS convention of logical to world transformation is that the spectral coordinate systems use both. In particular, for the more complex multispec systems the stored coefficients are defined only in terms of the physical coordinates, implying use of the logical to physical transformation first. Also a doppler factor is part of this WCS separate from the function coefficients. For a linear dispersion relation the simpler FITS WCS is used with the above convention that the FITS keywords refer to the combined logical-physical-world-doppler transformations.

There are some special MWCS keywords which are part of the WCS description. The keyword WCSDIM defines the dimensionality of the overall world coordinate system. Often this is the same as the image dimensionality but in the case of operations which dimensionally reduce the image, such as extracting a single line from a higher dimensional image, it is the dimensionality of the original image. The keyword WAXMAP01 is used for the case of dimensionally reduced images to indicate which axes are relevant and the constant value of the reduced axes. The syntax of this keyword are pairs of integer values, one for each original axis. The first number of each pair indicates which current *logical* axis corresponds to the original *physical* axis or zero if that axis is missing. When the first number is zero the second number gives the offset to the element of the original axis which is missing. As an example consider a three dimensional image in which the second plane is extracted (an IRAF image section of [*,2,*]). The keyword would then be `WAXMAP01 = '1 0 0 1 2 0'`.

In addition to individual FITS keywords the MWCS also stores information in keyword identified *attribute* strings. These attribute strings are stored as FITS keywords by collecting them into one very long string with the individual attributes beginning with the keyword, followed by an equal sign, and then the attribute string. If the attribute string contains whitespace it is quoted. The long string of attributes is then stored as a series of indexed FITS cards by breaking the string at the end of each card. This provides a maximally efficient storage of the arbitrarily long character string attributes. The FITS card keywords begin with the characters `WAT` followed by an axis number (0 applying to all axes), an underscore, and a sequence number. For examples see the accompanying figures.

Accessing the MWCS attributes from the FITS representation in an non-IRAF program can be a challenge since the strings may be arbitrarily long and may occur at any point in the set of keywords. Extracting a single attribute string by name from an OIF (original IRAF image format) in C or FORTRAN programs may be accomplished using an IRAF IMFORT routine available from the author. The user program must still parse the attribute string based on the information given below.

Some common attribute strings used by MWCS routines and the IRAF applications are the system name and the axes coordinate types, labels, units, and formats. Axis attributes may occur for each axis in the image. The axes coordinate types also appear in the FITS `CTYPE` keywords. The coordinate types tell the software how to interpret the WCS for that axis.

4. Linear Spectral World Coordinate Systems

When there is a linear or log-linear relation between pixels and dispersion coordinates which is the same for all spectra the external representation used is simple linear FITS. This applies to one, two, and three dimensional data. The higher dimensional data may have either linear spatial axes such as long slit or Fabry-Perot spectra or the equispec format where each one dimensional spectrum has the same dispersion.

The FITS image header keywords describing the spectral world coordinates are `CTYPE`i, `CRPIX`i, `CRVAL`i, `CDELT`i, and `CD`i_i where i is the axis number. Equations 2 and 3 define a wavelength in terms of these parameters for axis i and the logical coordinate l. The keyword `DC-FLAG` identifies the dispersion type by a value of 0 for linear or 1 for log-linear sampling. Note that though the coefficients are defined in log space the MWCS evaluates then as non-log wavelength. For spatial spectra there should also be a `DISPAXIS` parameter identifying the image axis, that is i, along which the dispersion runs. For equispec format the dispersion axis i is always 1.

$$\lambda = \mathtt{CRVAL}i + \mathtt{CD}i\_i \cdot (l - \mathtt{CRPIX}i) \qquad (2)$$

Figure 1. Long Slit Spectrum with Linear Dispersion Function

```
WAT0_001= 'system=world'
WAT1_001= 'wtype=linear'
WAT2_001= 'wtype=linear label=Wavelength units=Angstroms'
WCSDIM  =                    2
DISPAXIS=                    2
DC-FLAG =                    0

CTYPE1  = 'LINEAR  '
LTV1    =                 -10.
LTM1_1  =                   1.
CRPIX1  =                  -9.
CRVAL1  =     19.5743865966797
CD1_1   =     1.01503419876099

CTYPE2  = 'LINEAR  '
LTV2    =                -49.5
LTM2_2  =                  0.5
CRPIX2  =                 -49.
CRVAL2  =       4204.462890625
CD2_2   =     12.3337936401367
```

$$\lambda \quad = \quad 10^{\mathtt{CRVAL}i+\mathtt{CD}i\_i\cdot(l-\mathtt{CRPIX}i)} \tag{3}$$

Figure 1 shows the WCS keywords for a two dimensional long slit spectrum. The coordinate system is defined to be a generic *world* system and the `wtype` attributes and `CTYPE` keywords define the axes to be linear. The other attributes define a label and unit for the second axis which is the dispersion axis as indicated by the `DISPAXIS` keyword. The `LTM/LTV` keywords in this example show that a subsection of the original image has been extracted with a factor of 2 block averaging along the dispersion axis. The actual coordinates relative to the current image logical pixel coordinates are given by the standard FITS keywords.

Figure 2 shows the WCS keywords for a three dimensional image where each line is an independent spectrum or associated data but where all spectra have the same linear dispersion. This type of coordinate systems has the system name "equispec". The ancillary information about each aperture is found in the `APNUM` keywords. These give the aperture number, beam number, doppler correction, and extraction limits. These parameters were defined previously. Note that the doppler factor is a record of the dopper correction which is folded into the `CRVAL1/CDELT1/CD1_1` keywords. The `LTM/LTV` keywords have their default values; i.e. the logical and physical coordinates are the same.

5. Multispec Spectral World Coordinate System

The multispec spectral world coordinate systems apply only to one dimensional spectra; i.e. there is no analog for the spatial type spectra. They are used either when there are multiple 1D spectra with differing dispersion functions in a single image or when the dispersion functions are nonlinear.

Figure 2.   Spectra with Common Linear Dispersion Function

```
WAT0_001= 'system=equispec'
WAT1_001= 'wtype=linear label=Wavelength units=Angstroms'
WAT2_001= 'wtype=linear'
WAT3_001= 'wtype=linear'
WCSDIM  =                    3
DC-FLAG =                    0
APNUM1  = '41 3 0.1 7.37 13.48'
APNUM2  = '15 1 0.1 28.04 34.15'
APNUM3  = '33 2 0.1 43.20 49.32'

CTYPE1  = 'LINEAR  '
LTM1_1  =                   1.
CRPIX1  =                   1.
CRVAL1  =             4204.463
CD1_1   =     6.16689700000001

CTYPE2  = 'LINEAR  '
LTM2_2  =                   1.
CD2_2   =                   1.

CTYPE3  = 'LINEAR  '
LTM3_3  =                   1.
CD3_3   =                   1.
```

The multispec coordinate system is always two dimensional though there may be an independent third axis. The two axes are coupled and they both have axis type `multispec`. When the image is one dimensional the physical line is specified by the dimensional reduction keyword `WAXMAP01`. The second, line axis, has world coordinates of aperture number. The aperture numbers are integer values and need not be in any particular order but do need to be unique. This aspect of the WCS is not of particular user interest but applications use the inverse world to physical transformation to select a spectrum line given a specified aperture.

The dispersion functions are specified by attribute strings with the identifier *specN* where N is the *physical* image line. The attribute strings contain a series of numeric fields. The fields are indicated symbolically in (4) and (5).

$$\begin{aligned} specN &= ap\ beam\ dtype\ \lambda_1\ d\lambda\ n_\lambda\ z\ aplow\ aphigh\ [functions] \quad (4) \\ function_i &= w_i\ \Delta\lambda_i\ ftype\ [parameters]\ [coefficients] \quad (5) \end{aligned}$$

The first nine fields in the attribute are common to all the dispersion functions. The first field of the WCS attribute is the aperture number, the second field is the beam number, the seventh is a doppler factor, and the eighth and ninth fields are the aperture limits. These parameters were discussed earlier. The third field, *dtype*, is an integer code with the same function as `DC-FLAG`. A value of -1 indicates the coordinates are not dispersion coordinates (the spectrum is not dispersion calibrated), a value of 0 indicates linear dispersion, a value of 1 indicates log-linear dispersion, and a value of 2 indicates a nonlinear dispersion.

The next two fields are the dispersion coordinate of the first *physical* pixel and the average dispersion interval per *physical* pixel. For linear and log-linear dispersion types the dispersion is exact while for the nonlinear dispersion functions it is approximate. The next field is the number of valid pixels. It is possible to have spectra with varying lengths in the same image. In that case the image is as big as the biggest spectrum and the number of pixels selects the actual data in each image line.

Following these fields are zero or more function descriptions. For linear or log-linear dispersion coordinate systems there are no function fields. For the nonlinear dispersion systems the function fields specify a weight, a wavelength offset, the type of dispersion function, and the parameters and coefficients describing it. The function type codes, $ftype$, are 1 for a chebyshev polynomial, 2 for a legendre polynomial, 3 for a cubic spline, 4 for a linear spline, 5 for a pixel coordinate array, and 6 for a sampled coordinate array. The number of fields before the next function and the number of functions are determined from the parameters of the preceding function until the end of the attribute is reached.

Equation 6 shows how the final wavelength is computed based on the $nfunc$ individual dispersion functions $\Lambda(p)$. Note that this is completely general in that different functions types may be combined. However, in practice when multiple functions are used they are generally of the same type and represent a calibration before and after the actual object observation with the weights based on the relative time difference between the calibration dispersion functions and the object observation.

$$\lambda = \sum_{i=1}^{nfunc} w_i(\Delta\lambda_i + \Lambda_i(p))/(1+z) \tag{6}$$

The multispec coordinate systems define a transformation between pixel, p, and world coordinates, λ. Generally there is an intermediate coordinate system used. The following equations define these coordinates. Equation 7 shows the transformation between logical, l, and physical, p, coordinates based on the `LTM/LTV` keywords. The polynomial functions are defined in terms of a normalized coordinate, n, as shown in equation 8. The normalized coordinates run between -1 and 1 over the range of physical coordinates, p_{min} and p_{max} which are parameters of the function, upon which the coefficients were defined. The spline functions map the physical range into an index over the number of evenly divided spline pieces, $npieces$, which is a parameter of the function. This mapping is shown in equations 9 and 10 where s is the continuous spline coordinate and j is the nearest integer less than or equal to s.

$$\begin{aligned}
p &= (l - \mathtt{LTV1})/\mathtt{LTM1_1} && (7)\\
n &= (p - p_{middle})/(2 * p_{range}) \\
&= (p - (p_{max} + p_{min})/2)/(2 * (p_{max} - p_{min})) && (8)\\
s &= (p - p_{min})/(p_{max} - p_{min}) * npieces && (9)\\
j &= int(s) && (10)
\end{aligned}$$

Figure 3. Echelle Spectrum with Linear Dispersion Function

```
WAT0_001= 'system=multispec'
WAT1_001= 'wtype=multispec label=Wavelength units=Angstroms'
WAT2_001= 'wtype=multispec spec1 = "1 113 0 4955.44287109375 0.0568952970206737'
WAT2_002= '5 256 0. 23.22 31.27" spec2 = "2 112 0 4999.081054687501 0.063871018'
WAT2_003= '58854293 256 0. 46.09 58.44" spec3 = "3 111 0 5043.505859375 0.07096'
WAT2_004= '928358078002 256 0. 69.28 77.89"                                   '
WCSDIM  =                    2
DC-FLAG =                    0

CTYPE1  = 'MULTISPE'
LTM1_1  =                   1.
CD1_1   =                   1.

CTYPE2  = 'MULTISPE'
LTM2_2  =                   1.
CD2_2   =                   1.
```

5.1. Linear and Log Linear Dispersion Function

The linear and log-linear dispersion functions are described by a wavelength at the first *physical* pixel and a wavelength increment per *physical* pixel. A doppler correction may also be applied. Equations 11 and 12 show the two forms. Note that the coordinates returned are always wavelength even though the internal representation and the coefficient values may be log-linear.

$$\lambda = (\lambda_1 + d\lambda \cdot (p-1))/(1+z) \tag{11}$$
$$\lambda = 10^{(\lambda_1 + d\lambda \cdot (p-1))}/(1+z) \tag{12}$$

Figure 3 shows an example from a multispec image with independent linear dispersion coordinates. This is a linearized echelle spectrum where each order (identified by the beam number) is stored as a separate image line.

5.2. Chebyshev Polynomial Dispersion Function

The parameters for the chebyshev polynomial dispersion function are the *order* (number of coefficients) and the normalizing range of physical coordinates, p_{min} and p_{max}, over which the function is defined and which are used to compute n. Following the parameters are the *order* coefficients, c_i. Equation 13 shows how to evaluate the function using an iterative definition where $x_1 = 1$, $x_2 = n$, and $x_i = 2nx_{i-1} - x_{i-2}$.

$$\Lambda = \sum_{i=1}^{order} c_i x_i \tag{13}$$

5.3. Legendre Polynomial Dispersion Function

The parameters for the legendre polynomial dispersion function are the *order* (number of coefficients) and the normalizing range of physical coordinates, p_{min}

Figure 4. Echelle Spectrum with Legendre Polynomial Function

```
WAT0_001= 'system=multispec'
WAT1_001= 'wtype=multispec label=Wavelength units=Angstroms'
WAT2_001= 'wtype=multispec spec1 = "1 113 2 4955.442888635351 0.056896148108966'
WAT2_002= '83 256 0. 23.22 31.27 1. 0. 2 4 1. 256. 4963.016311209095 8.07119416'
WAT2_003= '976664 -0.3191636898579552 -0.8169352858733255" spec2 = "2 112 2 499'
WAT2_004= '9.081188912082 0.06387049476832223 256 0. 46.09 58.44 1. 0. 2 4 1. 2'
WAT2_005= '56. 5007.401409453303 8.555959076467951 -0.1767324582670318 -0.41247'
WAT2_006= '09935064388" spec3 = "3 111 2 5043.505764869474 0.07097050533145603 '
WAT2_007= '256 0. 69.28 77.89 1. 0. 2 4 1. 256. 5052.586239197408 9.04945848599'
WAT2_008= '271 -0.03173489817897474 -7.190562320405975E-4"                      '
WCSDIM  =                    2
DC-FLAG =                    0

CTYPE1  = 'MULTISPE'
LTM1_1  =                   1.
CD1_1   =                   1.

CTYPE2  = 'MULTISPE'
LTM2_2  =                   1.
CD2_2   =                   1.
```

and p_{max}, over which the function is defined and which are used to compute n. Following the parameters are the *order* coefficients, c_i. Equation 14 shows how to evaluate the function using an iterative definition where $x_1 = 1$, $x_2 = n$, and $x_i = ((2i-3)nx_{i-1} - (i-2)x_{i-2})/(i-1)$.

$$\Lambda = \sum_{i=1}^{order} c_i x_i \tag{14}$$

Figure 4 shows an example from a multispec image with independent non-linear dispersion coordinates. This is again from an echelle spectrum. Note that the IRAF **echelle** package determines a two dimensional dispersion function, in this case a bidimensional legendre polynomial, with the independent variables being the order number and the extracted pixel coordinate. To assign and store this function in the image is simply a matter of collapsing the two dimensional dispersion function by fixing the order number and combining all the terms with the same order.

5.4. Linear Spline Dispersion Function

The parameters for the linear spline dispersion function are the number of spline pieces, *npieces*, and the range of physical coordinates, p_{min} and p_{max}, over which the function is defined and which are used to compute the spline coordinate s. Following the parameters are the $npieces+1$ coefficients, c_i. The two coefficients used in a linear combination are selected based on the spline coordinate, where a and b are the fractions of the interval in the spline piece between the spline knots, $a = (j+1) - s$, $b = s - j$, and $x_0 = a$, and $x_1 = b$.

$$\Lambda = \sum_{i=0}^{1} c_{i+j} x_i \tag{15}$$

5.5. Cubic Spline Dispersion Function

The parameters for the cubic spline dispersion function are the number of spline pieces, *npieces*, and the range of physical coordinates, p_{min} and p_{max}, over which the function is defined and which are used to compute the spline coordinate s. Following the parameters are the $npieces+3$ coefficients, c_i. The four coefficients used are selected based on the spline coordinate. The fractions of the interval between the integer spline knots are given by a and b, $a = (j+1) - s$, $b = s - j$, and $x_0 = a^3$, $x_1 = (1 + 3a(1 + ab))$, $x_2 = (1 + 3b(1 + ab))$, and $x_3 = b^3$.

$$\Lambda = \sum_{i=0}^{3} c_{i+j} x_i \tag{16}$$

5.6. Pixel Array Dispersion Function

The parameters for the pixel array dispersion function consists of just the number of coordinates *ncoords*. Following this are the wavelengths at integer physical pixel coordinates starting with 1. To evaluate a wavelength at some physical coordinate, not necessarily an integer, a linear interpolation is used between the nearest integer physical coordinates and the desired physical coordinate where a and b are the usual fractional intervals $k = int(p)$, $a = (k+1) - p$, $b = p - k$ and $x_0 = a$, and $x_1 = b$.

$$\Lambda = \sum_{i=0}^{1} c_{i+j} x_i \tag{17}$$

5.7. Sampled Array Dispersion Function

The parameters for the sampled array dispersion function consists of the number of coordinate pairs, *ncoords*, and a dummy field. Following these are the physical coordinate and wavelength pairs which are in increasing order. The nearest physical coordinates to the desired physical coordinate are located and a linear interpolation is computed between the two sample points.

6. Future Developments

The complexity of the multispec format is due to the need to pack multiple one dimensional spectra in a single image and to encode the nonlinear functions in FITS keywords. For the IRAF disk spectral formats the solution will be more flexible formats which allow collecting multiple objects (one dimensional spectra) in a single file and provide general storage of large character and numeric arrays rather than the long character string attributes packed in FITS cards.

This will not solve the problem of transport using the FITS standard. The FITS standard still needs to address the question of general nonlinear dispersion coordinate functions. The WCS presented here provides only an interim solution.

When a standard is defined it can be expected that the IRAF WCS would be represented in that way.

In the shorter term some other ideas may be explored. Some ideas are to use a group format, such as that used with Hubble Space Telescope data, and to add support for spectra as tables with dispersion/flux columns.

References

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Tody, D., 1989, *Mini-WCS Interface*, NOAO/CCS, P.O. Box 26732, Tucson, AZ 85726